Kummer strikes back: new DH speed records

Chitchanok Chuengsatiansup

9th December 2014

Joint work with Daniel J. Bernstein, Tanja Lange, and Peter Schwabe

Diffie–Hellman Key Exchange

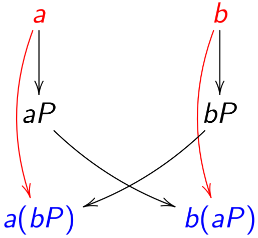
a b

Diffie–Hellman Key Exchange

$a \longrightarrow aP$

$b \longrightarrow bP$

Diffie–Hellman Key Exchange



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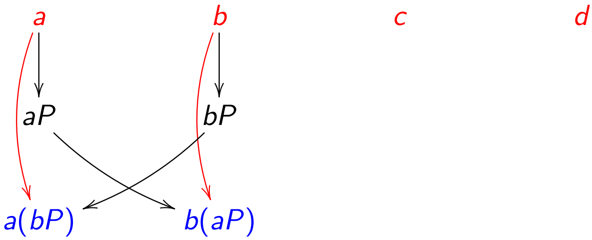

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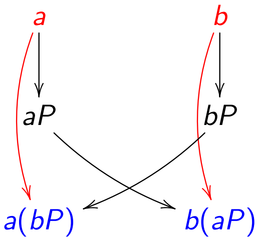

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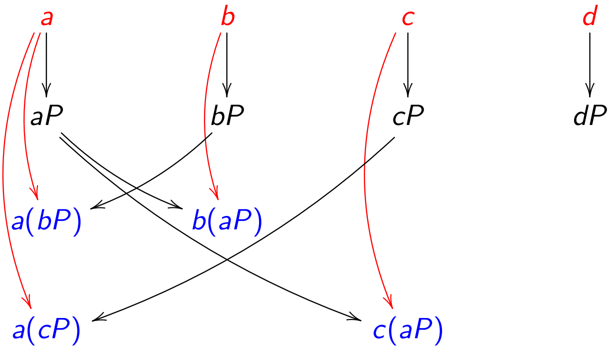

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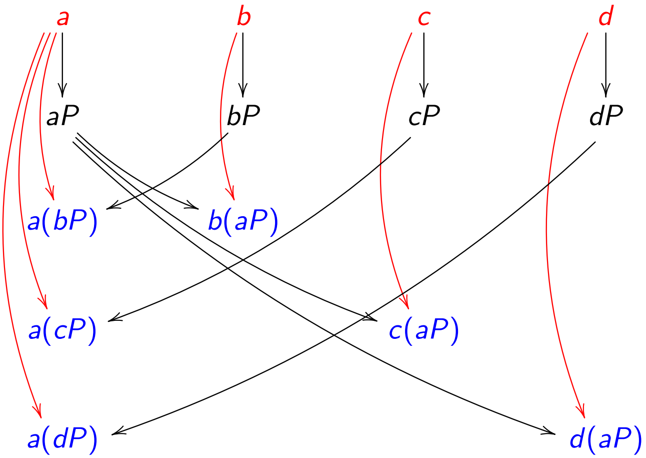

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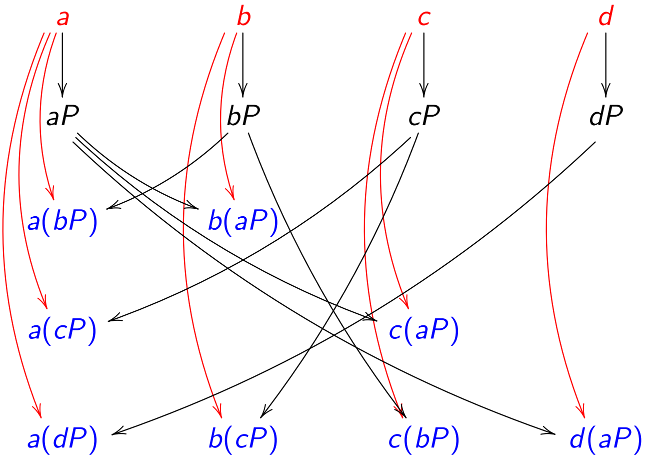

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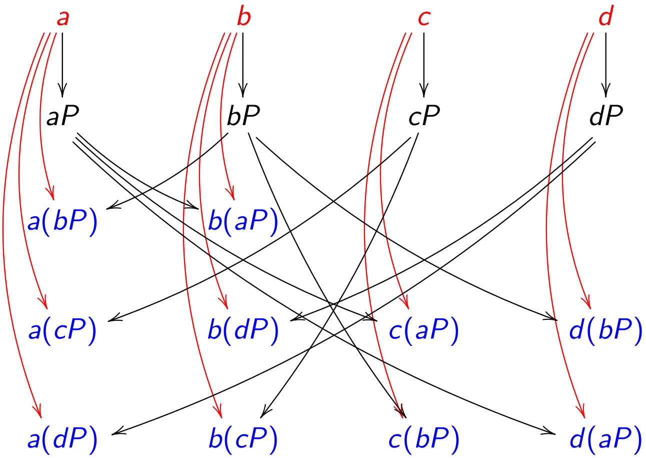

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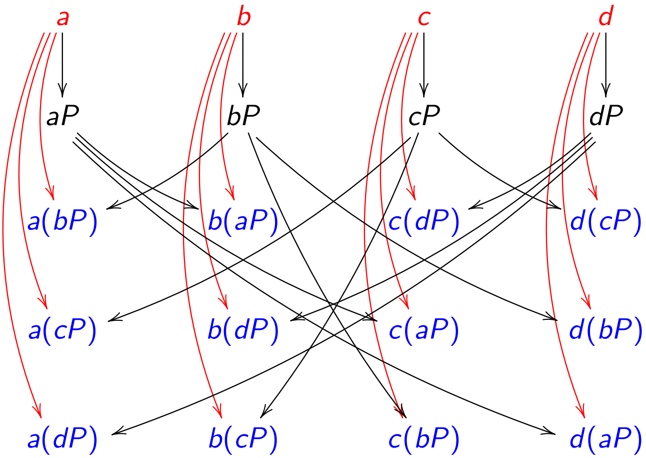

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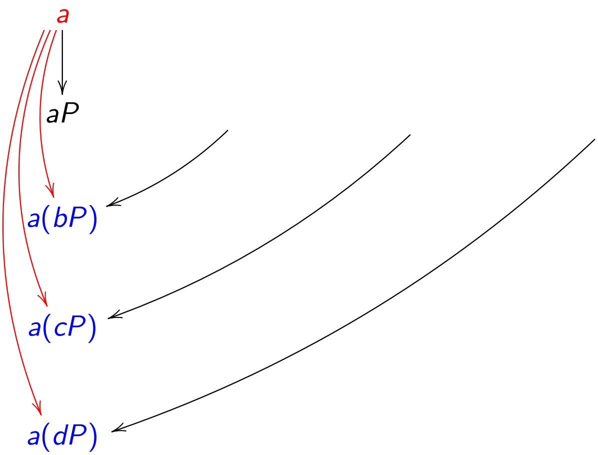
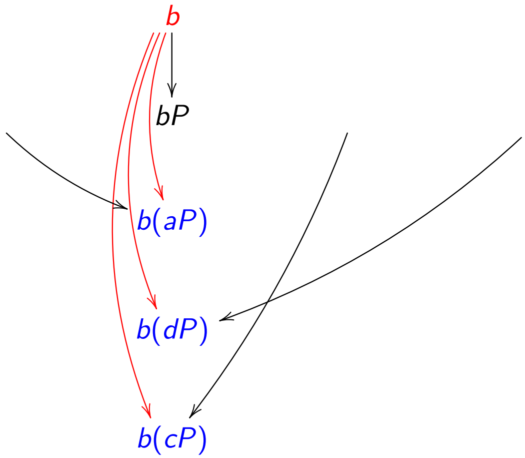


main DH challenge: make variable-base scalar mult as fast as possible



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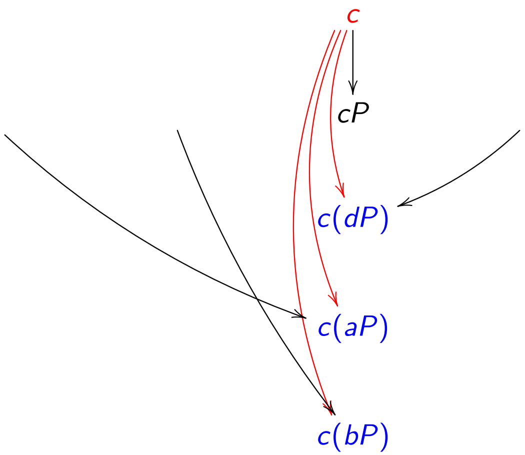

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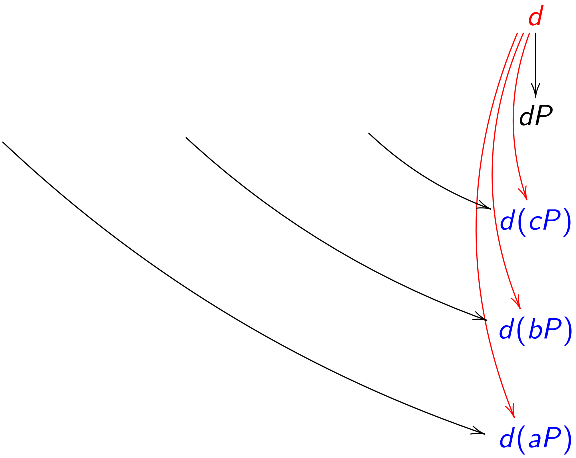


main DH challenge: make variable-base scalar mult as fast as possible

Ladder

Input: $Q, n = (n_{i-1}, \ldots, n_0)_2$
Output: $R_0 = nQ$

$R_0 \leftarrow 0Q; \quad R_1 \leftarrow 1Q$
if $n_i = 0$ **then**
$\quad R_0 \leftarrow 2R_0; \quad R_1 \leftarrow R_0 + R_1$
else
$\quad R_0 \leftarrow R_0 + R_1; \quad R_1 \leftarrow 2R_1$

Return R_0

Example: Compute $9Q$

$9 = 1001_2$

$(R_0, R_1) = (0Q, 1Q)$

$n_3 = 1 : \quad (1Q, 2Q)$

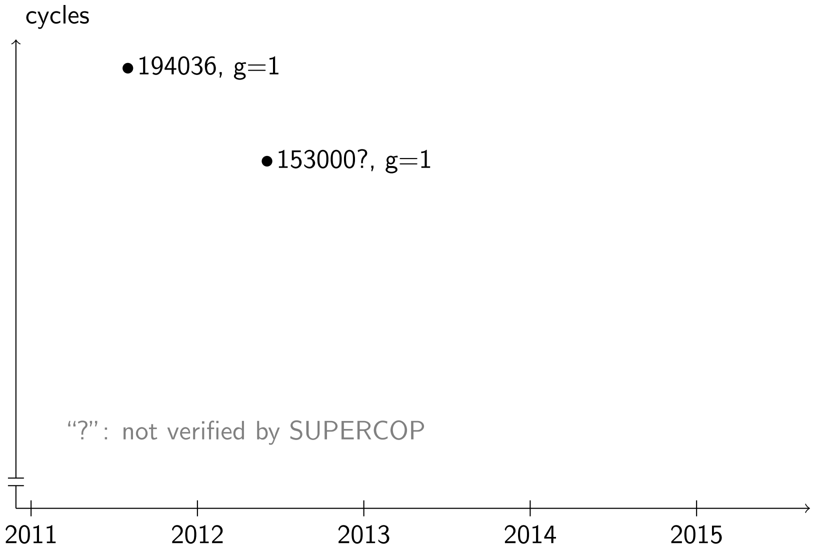
$n_2 = 0 : \quad (2Q, 3Q)$

$n_1 = 0 : \quad (4Q, 5Q)$

$n_0 = 1 : \quad (9Q, 10Q)$

Elliptic VS Hyperelliptic

Sandy Bridge at high security level and (claimed) constant time



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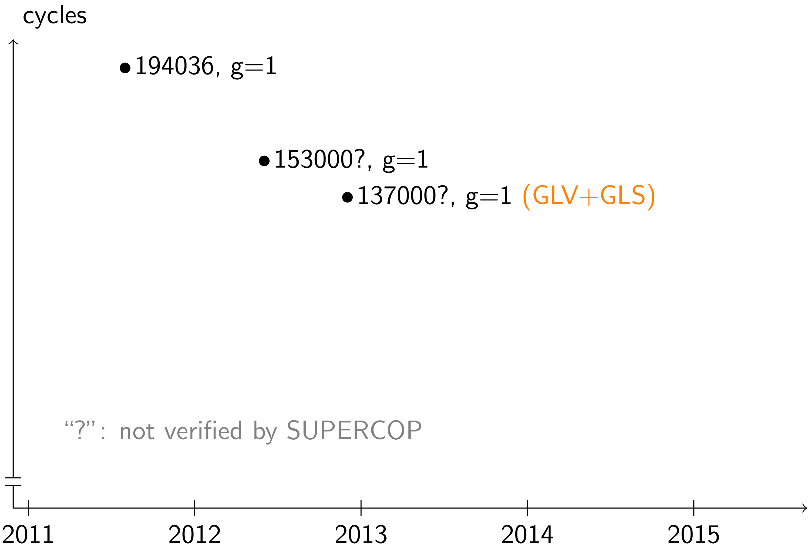

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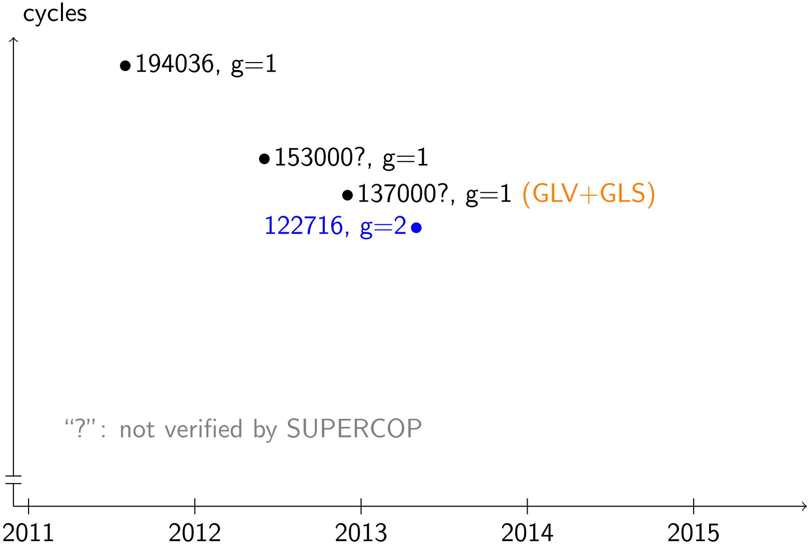

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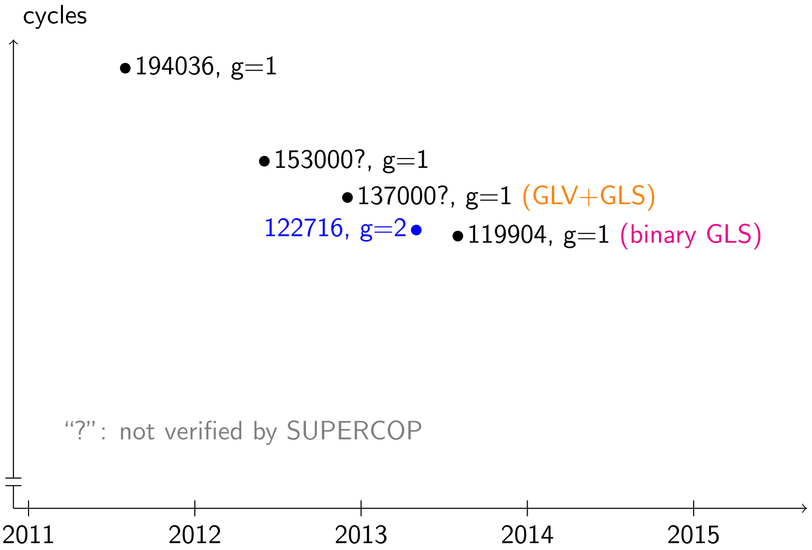

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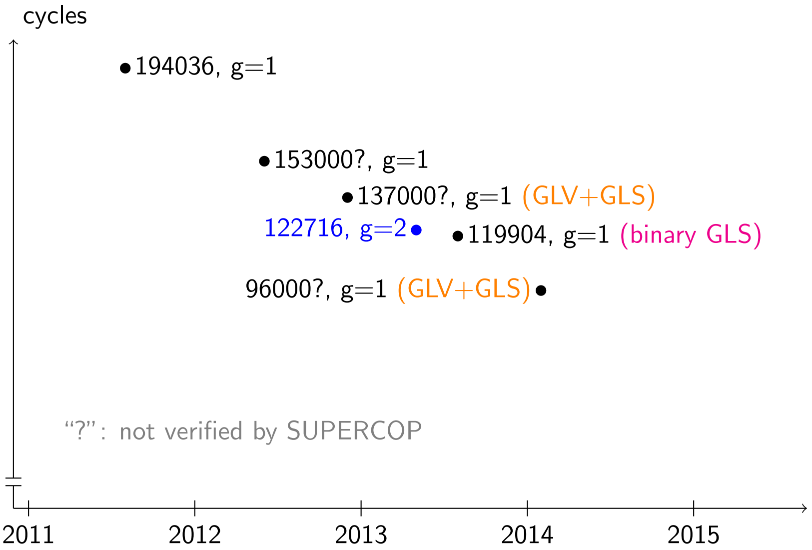

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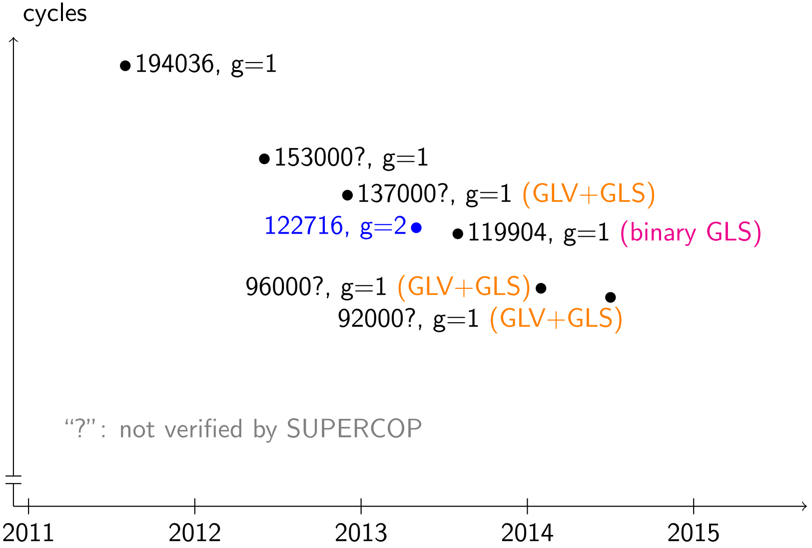

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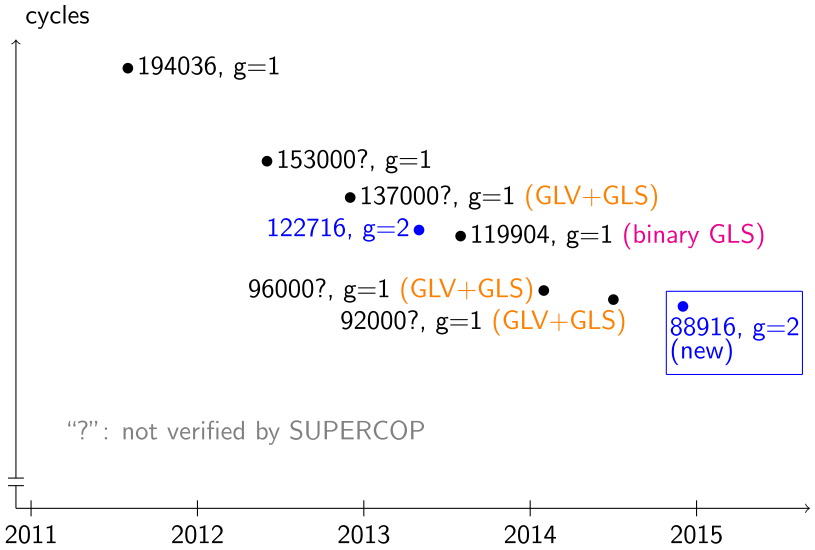
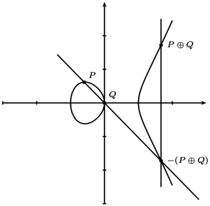


Elliptic-Hyperelliptic Analogy

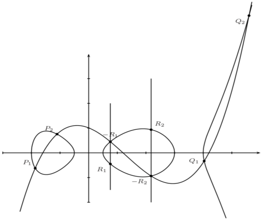
ECC



$$y^2 = x^3 + ax + b$$

$\longrightarrow$ x-line represented as $(X : Z)$

HECC



$$v^2 = u^5 + f_4 u^4 + f_3 u^3 + f_2 u^2 + f_1 u^1 + f_0$$

$\longrightarrow$ Kummer surface represented as $(X : Y : Z : T)$

Hyperelliptic Advantages

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- new: formulas well suited for vectorization

Vectorization

without vector

a

$+$

b

$=$

$a + b$

Vectorization

without vector

a
$+$
b
$=$
$a+b$

with vector

a_0	a_1	a_2	a_3
$+$	$+$	$+$	$+$
b_0	b_1	b_2	b_3
$=$	$=$	$=$	$=$
a_0+b_0	a_1+b_1	a_2+b_2	a_3+b_3

Vectorization

without vector

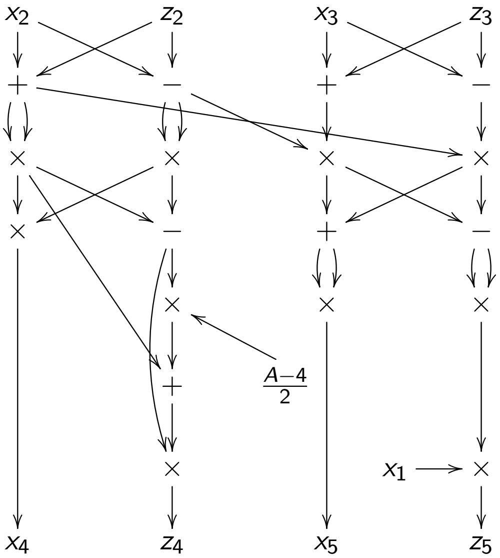
a
$+$
b
$=$
$a + b$

with vector

a_0	a_1	a_2	a_3
$+$	$+$	$+$	$+$
b_0	b_1	b_2	b_3
$=$	$=$	$=$	$=$
$a_0 + b_0$	$a_1 + b_1$	$a_2 + b_2$	$a_3 + b_3$

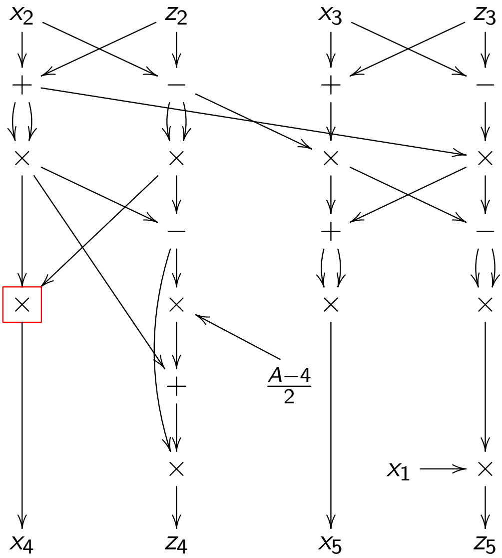
- **single** instruction performing n **independent** operations on **aligned** inputs

ECC Montgomery Ladder (original)



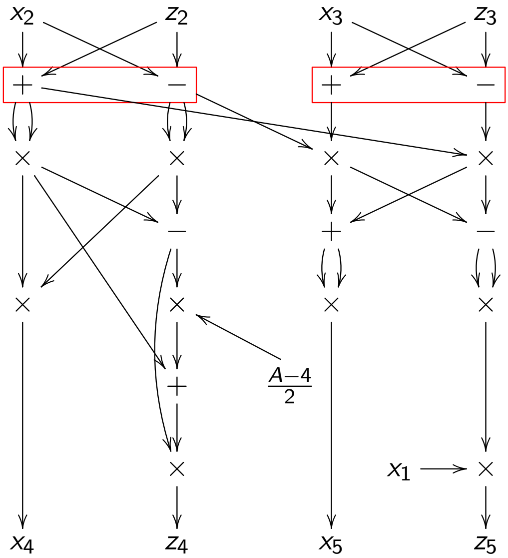
ECC Montgomery Ladder (2-way vectorization)

- move $\times$ down



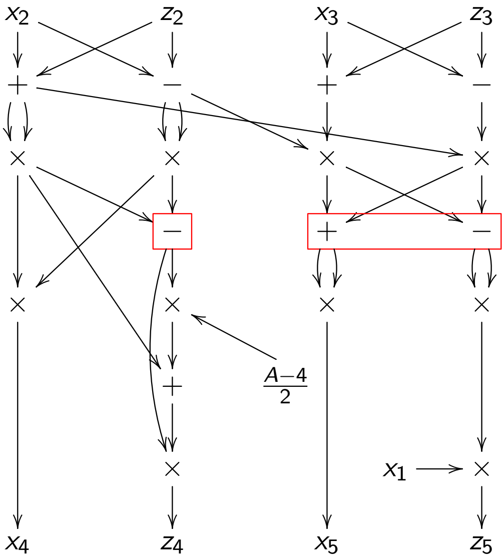
ECC Montgomery Ladder (2-way vectorization)

- require permutation
- move × down



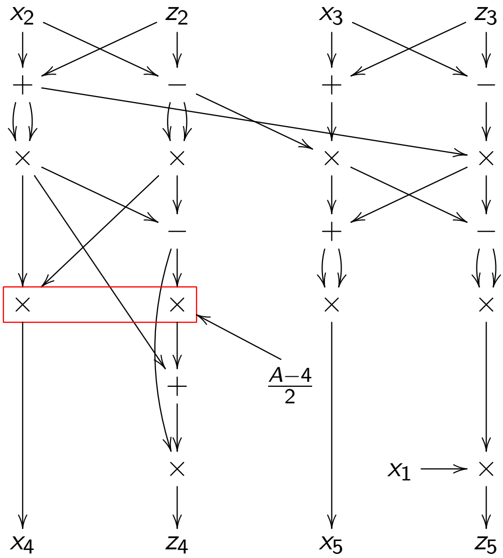
ECC Montgomery Ladder (2-way vectorization)

- require permutation
- move $\times$ down
- require permutation and leave $+$ idle



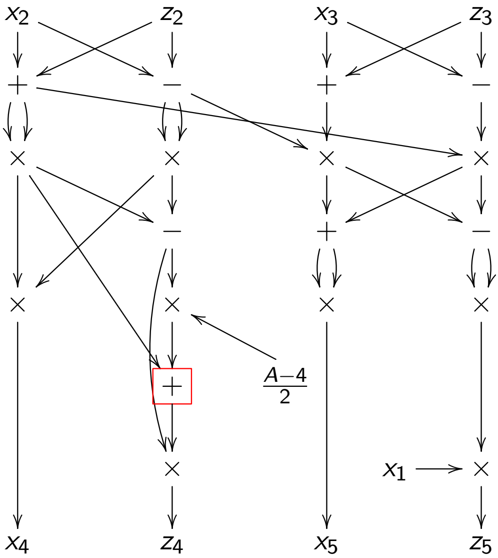
ECC Montgomery Ladder (2-way vectorization)

- require permutation
- move × down
- require permutation and leave + idle
- slow down mult. by constant



ECC Montgomery Ladder (2-way vectorization)

- require permutation
- move × down
- require permutation and leave + idle
- slow down mult. by constant
- nothing to match +



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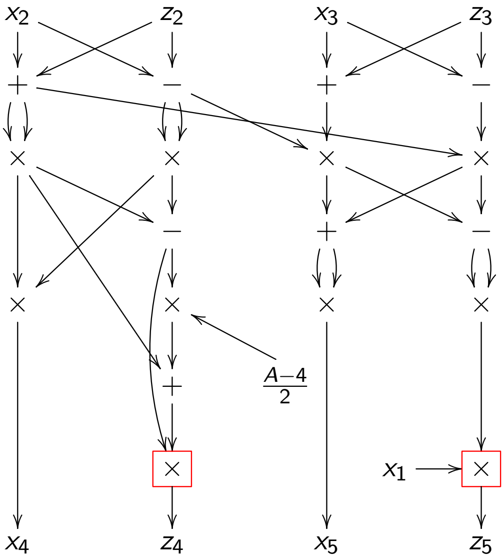
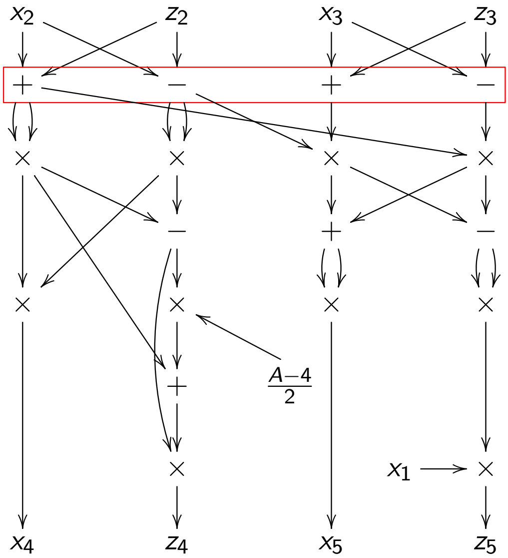


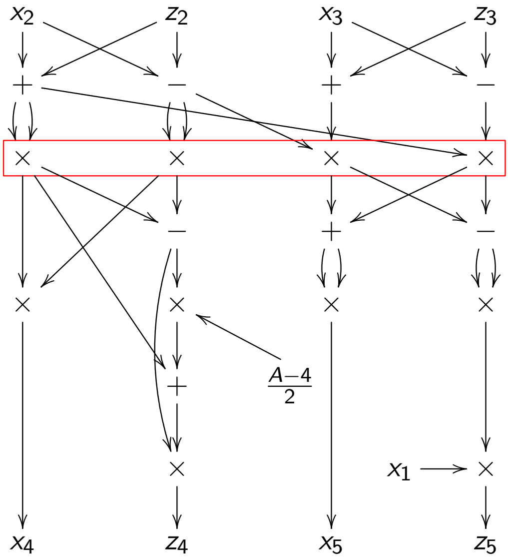
ECC Montgomery Ladder (4-way vectorization)

- match + with −



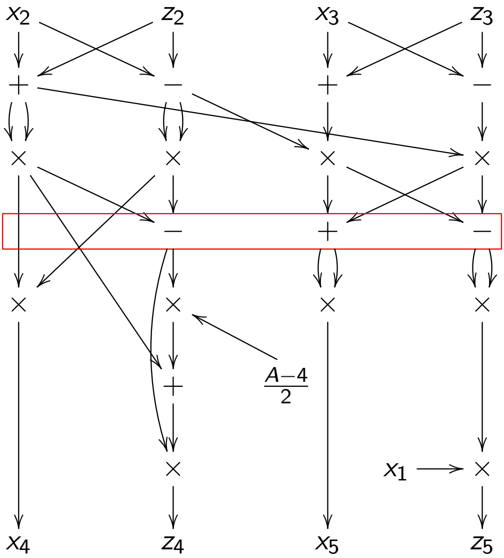
ECC Montgomery Ladder (4-way vectorization)

- match + with −
- slow down squaring

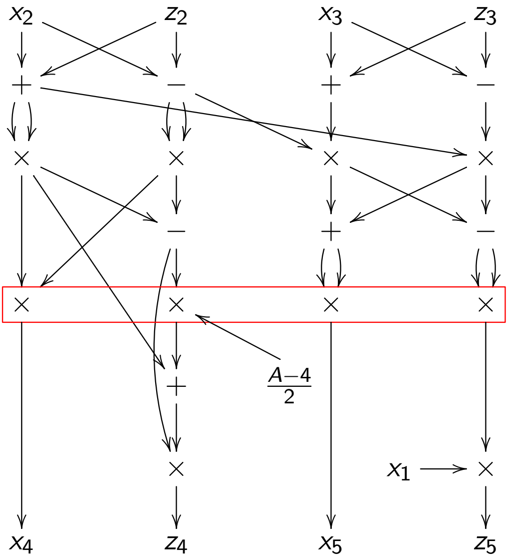


ECC Montgomery Ladder (4-way vectorization)

- match + with −
- slow down squaring
- nothing to match and match + and −



- match + with −
- slow down squaring
- nothing to match and match + and −
- slow down mult. by constant and squaring



ECC Montgomery Ladder (4-way vectorization)

- match $+$ with $-$
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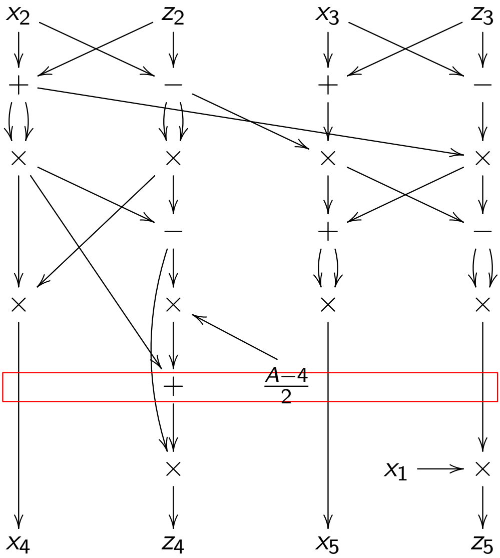
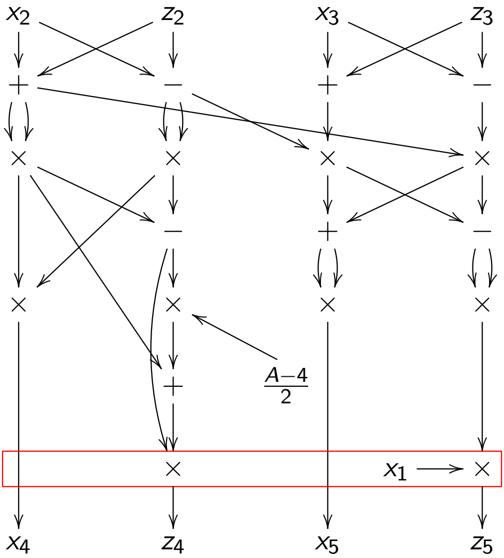
- nothing to match $+$

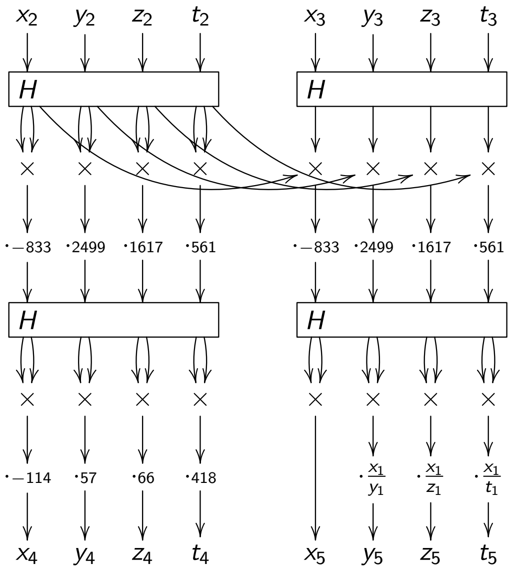


ECC Montgomery Ladder (4-way vectorization)

- match $+$ with $-$
- slow down squaring
- nothing to match and match $+$ and $-$
- slow down mult. by constant and squaring
- nothing to match $+$
- nothing to match $\times$



Squared Kummer Surface Ladder



Making It Run Really Fast

- maximize usage of available vector multipliers
- minimize cost from carries
  - use redundant representation
  - use non-integer radix, e.g., $2^{25.4}$ on Cortex-A8
  - do not perform full carry
  - do parallel carry chain
- eliminate redundancy inside field operations
  - precompute to reuse values $2f_1, 2f_2, 2f_3, 2f_4$
- minimize overhead from permutations
  - organize data to fit instruction format
- minimize permutations in H
  - use different order of input/output and change sign
  - e.g. we reduced 144 Sandy Bridge permutations to 36
- schedule instructions to keep CPU as busy as possible
- see paper for details

Fastest Diffie–Hellman Ever!!!

Arch	Cycles	g	Field	Source of software	
A8-slow	497389	1	$2^{255}-19$	BeSc	CHES 2012
A8-slow	305395	2	$2^{127}-1$	**new (this paper)**	
A8-fast	460200	1	$2^{255}-19$	BeSc	CHES 2012
A8-fast	273349	2	$2^{127}-1$	**new (this paper)**	
Sandy	194036	1	$2^{255}-19$	BeDuLaScYa	CHES 2011
Sandy	153000?	1	$2^{252}-2^{232}-1$	Hamburg	
Sandy	137000?	1	$(2^{127}-5997)^2$	LoSi	Asiacrypt 2012
Sandy	122716	2	$2^{127}-1$	BoCoHiLa	Eurocrypt 2013
Sandy	119904	1	2^{254}	OlLóArRo	CHES 2013
Sandy	96000?	1	$(2^{127}-5997)^2$	FaLoSá	CT-RSA 2014
Sandy	92000?	1	$(2^{127}-5997)^2$	FaLoSá	July 2014
Sandy	88916	2	$2^{127}-1$	**new (this paper)**	
Haswell	161648	1	$2^{255}-19$	BeDuLaScYa	CHES 2011
Haswell	110740	2	$2^{127}-1$	BoCoHiLa	Eurocrypt 2013
Haswell	61712	1	2^{254}	OlLóArRo	CHES 2013
Haswell	60556	2	$2^{127}-1$	**new (this paper)**	